ALGEBRAIC TOPOLOGY (FALL 2016)

Course title: Algebraic Topology

Lecturer: Prof.dr.ir. Jan Draisma (jan.draisma@math.unibe.ch), and exercise classes by Olivier Mila (olivier.mila@math.unibe.ch).

Dates and venue: Wednesdays, 08-10 h (actually, lectures start 15 min. later), B6 (ExWi), and Friday, 08 - 10 h, B7 (ExWi). Start: Wednesday, September 21, 2016. The second hour on Fridays is for exercises.

Introduction. In algebraic topology we associate algebraic structures such as groups, rings, finite sets, etc. to topological spaces. We do so in a functorial manner: continuous maps between topological spaces lead to homomorphisms of the algebraic structures. These algebraic structures can be used to distinguish topological spaces, obtain information about fixed points of continuous self-maps, *etc.*

Algebraic topology has many applications in other areas, such as combinatorics, game theory, and more recently, analysis of geometric data. In addition to developing the abstract theory, we will also see some of these applications.

Requirements. I'm assuming knowledge of point-set topology, and also that you have already seen fundamental groups and covering spaces. I will re-define these, but go rather quickly through some of their properties, to then slow down at Seifert-van Kampen. I'm also assuming basic knowledge of groups (e.g., the classification of finitely generated Abelian groups) and rings.

Contents. The chapter numbers below refer to [Rot88].

- **Motivation (Chapter 0):** Brouwer's fixed-point theorem, invariance of domain, Nash equilibria [Nas51], point clouds [Car09, Ghr08].
- **Categories (Chapter 0):** definitions and examples.
- **Homotopy (Chapter 1):** definition, category of homotopy classes of topological spaces.
- **Fundamental group (parts of Chapters 3,7,10):** definition, relation to covering spaces, fundamental group of circle, amalgamated product of groups, Seifert-van Kampen theorem, CW-complexes, attaching cells. Topological groups have Abelian fundamental groups.
- **Singular homology (Chapters 2,4,5,6):** definition, pairs of topological spaces, verification of the Eilenberg-Steenrod properties (dimension, homotopy, excision, additivity, long exact sequences). Relation to the fundamental group. Applications.
- **Simplicial homology (Chapter 7):** definition, simplicial approximation, comparison with singular homology—simplicial homology is computable!
- **Application of topology to data science:** see [Car09].
- **Cohomology (Chapter 12):** definition, cup product, and if time allows Poincaré duality for compact, oriented, manifolds [Hat02].

Credits: 6ECTS. To participate in the final exam you must work out at least 9 of the weekly homework sheets. These are assigned on Wednesdays, handed in the next Wednesday, and discussed on Fridays.

Literature. We will follow mostly [Rot88], though for some proofs, exercises, and inspiration we will also use [Hat02]. The former is more algebraic in nature, the latter, more geometric.

References

[Car09] Gunnar Carlsson. Topology and data. *Bull. Am. Math. Soc., New Ser.*, 46(2):255–308, 2009.

[Ghr08] Robert Ghrist. Barcodes: The persistent topology of data. *Bull. Am. Math. Soc., New Ser.*, 45(1):61–75, 2008.

[Hat02] Allen Hatcher. *Algebraic topology.* Cambridge: Cambridge University Press, 2002. Freely available online at https://www.math.cornell.edu/~hatcher/AT/AT.pdf.

[Nas51] John Nash. Non-cooperative games. *Ann. Math. (2)*, 54:286–295, 1951.

[Rot88] Joseph J. Rotman. *An introduction to algebraic topology.* New York (FRG) etc.: Springer-Verlag, 1988.